

# Parametric Robustness Analysis for Cassini Spacecraft Using The LMI Approach without Frequency Sweep<sup>1</sup>

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## Abstract

A generalized Popov multiplier theory enables one to analyze system robustness with mixed real/complex parameters, repeated uncertainties, and multi-dimensional block uncertainties. Using the Cassini spacecraft thrust vector control (TVC) system as a design case study, modern robust control theory has been applied to the real flight project.

## 1. Introduction

The Cassini spacecraft will be visiting Saturn, one of the most interesting planets in our solar system, in the year 2004. The Cassini spacecraft will be launched in 1997 and achieve a Saturn orbit in 2004, leading to a four-year mission of orbiting Saturn and flying by its largest moon Titan. Scientists believe the Titan moon contains materials similar to primitive earth of millions of years ago.

In Cassini, there are three major attitude control subsystems: Thrust Vector Control (TVC) controls the AV burn maneuvers via the articulated main engines and Z-axis facing thrusters; Reaction Wheel Assembly (RWA) controls the fine pointing of the camera on science images; and the Reaction Control Subsystem (RCS) controls the large and quick angular maneuvers via a set of Y/Z axes facing thrusters.

Robust control analysis plays an important role in the design of spacecraft attitude control systems for ensuring satisfactory performance with sufficient stability margin. This paper focuses on the

analysis of the Cassini TVC system against uncertain fuel slosh and flexible boom dynamics. If the result of robustness analysis indicates that some uncertain parameters can threaten the stability of the attitude control system, robust synthesis tools can be invoked to design a better controller.

The latest tools developed in [7, 3, 9, 5] have been used to analyze robustness for Cassini and to compute the multivariable stability margin (MSM). First, the uncertain plant parameters such as fuel slosh and mag boom mode frequencies, mass properties, etc. will be pulled out of the closed-loop system. Then a guaranteed lower bound of the MSM will be computed based on the generalized Popov multiplier theory. Unlike the traditional structured singular value approach, this frequency dependent generalized Popov multiplier theory cannot only compute the MSM with respect to mixed and repeated real/complex uncertainties but can also avoid the time consuming frequency sweep that conventional methods always suffer [4]. It turns out that this Popov multiplier approach is at variance with the recently developed *Linear Matrix Inequality* (LMI) theory [5, 1].

## 2. Multiplier Approach for Robustness Analysis

The small-gain theorem and its associated *analysis* and *synthesis* design methods have been the most important breakthrough in robust control theory and application during the 80's. The issues related to analyzing the MSM of a given closed-loop system have been solved in a mathematically sound fashion ([6, 4]) and have been continuously improved by various approaches over the years. Basically, one can find the upper bounds of the

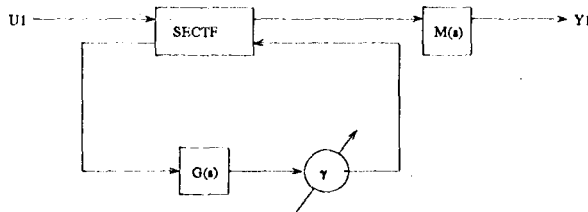
<sup>1</sup> Invited Session on Robust Control Design, American Control Conference, Baltimore, MO, June 29- July 1, 1994.

So-called  $K_m$  (or  $\mu$ ) function to access the system MSM against the given uncertainty bounds (see [2] and the references therein). However, depending on the nature of the uncertainty (i.e., real parameter uncertainty or frequency dependent complex norm bounded transfer functions), bounds such as the singular value or structured singular value (SSV) are too conservative for real parametric robustness. "Tighter and more accurate bounds of  $K_m$  are now available for mixed real/complex robustness analysis ([3, 7, 9, 5]).

The sector bilinear transform and Popov multiplier theory together play a crucial role in the overall methodology. The robustness analysis problem formulation is rather simple (ref. Figure 2):

$$\max_{\gamma, M \in \mathcal{M}} \text{Herm}\{M(j\omega) \text{Sect}f(\gamma G)\} > 0 \quad \forall \omega \quad (1)$$

This means that we want to find the greatest "real" number  $\gamma$  such that for some generalized Popov multiplier  $M(s)$ , the sector transformed transfer function (seen by the uncertainty) is strictly positive real. If such a multiplier  $M(s)$  exists, that real number " $\gamma$ " is the multivariable stability margin (MSM) of the system.



**Figure 1: The Robustness Analysis Problem Setup.**

The procedure of finding such an MSM is the following:

- Step 1: Assign a nominal value to  $\gamma$  (say 1 to start with) and normalize  $\|\gamma G\|_{\infty} = 1$
- Step 2: Sector transform the scaled quantity  $\gamma G$  from  $(-1, 1)$  to positive real sector  $(0, \infty)$
- Step 3: Using finite dimensional optimization approaches such as ellipsoid, cutting plane, interior point algorithms to find the optimal multiplier  $M(s)$  that can make the system positive real
- Step 4: If  $M(s)$  successfully found, go back to Step 1 and increase  $\gamma$ ; if not, go back to Step 1 decrease  $\gamma$ .

This type of  $\gamma$  iteration can be carried out using the standard golden section (binary search) method. Notice that this approach can easily be expanded to handle the following cases:

1. mixed real/complex uncertainties
2. repeated real/complex uncertainties
3. multi-dimensional uncertainty blocks

Another unique feature is that this method of finding the frequency dependent fixed order multiplier  $M(s)$  essentially eliminates the unreliable frequency sweep approach of the Structured Singular Value ( $\mu$ ) and the early version of the multiplier approach [8]. It gives one a guaranteed assessment of the MSM without missing any frequency points. Detailed definitions, terminologies, background theory, and proofs can be found in [3, 7, 9] which are omitted here.

This generalized Popov multiplier has a connection with the latest robust control theory: Linear Matrix Inequality (LMI). The background theory can be found in author's paper [5], which will not be repeated here.

### 3. Robustness Analysis of Cassini TVC Control System

The primary function of the Cassini TVC control system is to control the spacecraft AV burn maneuvers via the articulated main engines and thrusters. Detailed controller design and background can be found in [10]. The present paper focuses on the robustness analysis of the given TVC closed loop design.

Several uncertain plant parameters are of interest here:

1. Engine off-set
2. Spacecraft inertia
3. Fuel slosh mode frequency
4. Magboom structure mode frequency

Figure 3 shows the Simulink block diagram and the details of how to pull out a particular uncertainty. The input/output ports 1 to 6 are the multiplicative uncertainties associated with the parameters. Evaluating the "size" of the transfer function matrix (port # 1 to # 6) reveals the MSM of these uncertain parameters.



#### 4. Conclusion

The robustness of the Cassini Thrust Vector Control system was successfully analyzed using the latest tools developed under these-called generalized Popov multiplier theory. The results indicate that under the nominal expected damping levels, the control design achieves 15 % robustness margin. Even under worst-case zero damping conditions, a 3.5 % simultaneous robustness margin is achieved.

Unlike the conventional analysis methods, this new approach enables one to analyze multiple types of uncertainties such as mixed real/complex, repeated blocks or patterns, and even multi-dimensional combinations. The overall method is coded in MATLAB and provides an effective new tool for analyzing control system robustness.

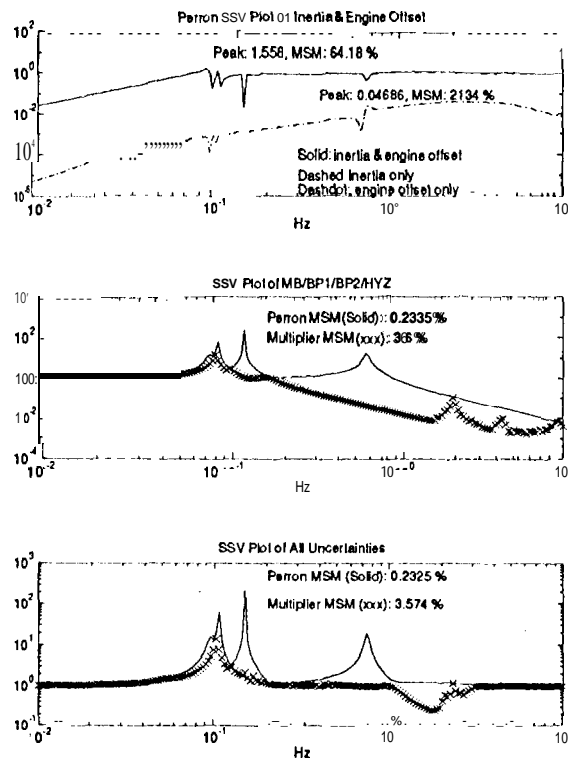


Figure 4: Frequency Sweep Approach (Perron SSV and Multiplier).

#### 5. Acknowledgment

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#### References

- [1] S. Boyd, V. Balakrishnan, E. Feron, and I. ElGhaoui, "Control System Analysis and Synthesis via Linear Matrix Inequality," *Proc. of American Control Conf.*, pp. 2147-2154, June 2-4, San Francisco, CA.
- [2] R. Y. Chiang, and M. G. Safonov, *Robust Control Toolbox*, The MathWorks inc., 1988-1994.
- [3] R. Y. Chiang, and M. G. Safonov, "Real  $K_m$  Synthesis via Generalized Popov Multiplier," *Proc. of American Control Conf.*, Chicago, IL, June, 1992.
- [4] J. C. Doyle, "Analysis of Feedback Systems with Structured Uncertainty," *IEEE Proc.*, vol. 129, pt. D, no. 6, pp. 242-250, 1982.
- [5] Jason Ly, M. G. Safonov, and R. Y. Chiang, "Real/Complex Multivariable Stability Margin Computation via Generalized Popov Multiplier - LMI Approach," *Proc. of 1994 American Control Conf.*, Baltimore, MD, June 29- July 1, 1994.
- [6] M. G. Safonov, "Stability Margins of Diagonally Perturbed Multivariable Feedback Systems," *IEEE Proc.*, vol. 129, pt. D, no. 6, pp. 252-255, 1982.
- [7] M. G. Safonov, and R. Y. Chiang, "Real/Complex  $K_m$ -Synthesis without Curve Fitting, in *Control and Dynamic Systems*, Academic Press 1993.
- [8] M. G. Safonov, and P. H. Lee, "(A Multiplier Method for Computing Real Multivariable Stability Margin," *Proc. of IFA C World Congress*, Sydney, Australia, July 1993.
- [9] M. G. Safonov, Jason Ly, and R. Y. Chiang, "p-Synthesis Robust Control: What's Wrong and How to Fix It?," *Proc. of Aerospace Control Systems*, pp. 563-568, Thousand Oaks, CA, May 1993.
- [10] P. J. Enright, "Thrust Vector Control Algorithm Design for the Cassini Spacecraft," *AIAA AAS/ASEE Aerospace Design Conference*, AIAA 93-1043, Irvine, Ca., February 16-19, 1993.